Solving Problems with the Work–Kinetic Energy Theorem

Purpose and Expected Outcome

Like the Impulse–Momentum Theorem, the Work–Kinetic Energy Theorem is a useful problem-solving principle. In this activity, you will learn how to apply the Work–Kinetic Energy Theorem to different situations. You will learn how to relate the forces exerted on an object to its displacement and changes in its speed.

Prior Experience / Knowledge Needed

You should know the definitions of work and kinetic energy, and be able to apply them to physical situations. You should be familiar with the Work–Kinetic Energy Theorem. Namely,

$$W_{\text{total}} = \Delta E_{K,\text{total}} \qquad \textbf{Work–Kinetic Energy Theorem}$$

For a rigid, non-rotating object, we get:

$$\begin{aligned} W_{\text{total}} &= \Delta\left(\tfrac{1}{2}Mv^2\right) \qquad \text{(for a rigid, non-rotating body of total mass } M\text{)} \\ &= \tfrac{1}{2}M(v_f)^2 - \tfrac{1}{2}M(v_i)^2 \end{aligned}$$

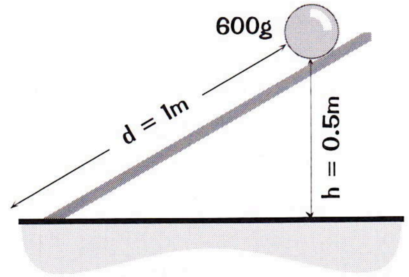
where W_{total} is the sum of the work done by all the forces exerted on the object.

Explanation of Activity and Example

For each of the problems below, first (a) indicate how you will use the definition of work, the definition of kinetic energy, and the Work–Kinetic Energy Theorem to solve for the desired unknown. Then (b) solve the problem and compute the value of the desired unknown. Use g = 10N/kg throughout.

Example. A 600g ball rolls down a 1m ramp inclined at 30° as shown.

What is the kinetic energy of the ball when it reaches the bottom?



***Answer*:**

The forces on the ball are (1) a normal force exerted by the incline, (2) a static friction force exerted by the incline, and (3) a gravitational force exerted by the earth. The work done by the normal force is zero, because it always points perpendicular to the displacement. The work done by the static friction force is zero, because the force is exerted through zero displacement. The work done by the gravitational force is positive and equal to the force of gravitation multiplied by the vertical component of the displacement. The total work is equal to the change in kinetic energy. We know the initial kinetic energy (zero), so we can find the final kinetic energy.

The solution is shown below. The Work–Kinetic Energy Theorem states:

$$W_{\text{total}} = \Delta E_{K,\text{total}}$$

Using the definitions of total work and change in kinetic energy, we get:

$$W_{\text{normal}} + W_{\text{static friction}} + W_{\text{gravitation}} = E_{K,f} - E_{K,i}$$

Using the definitions of work and kinetic energy, we get:

$$(0) + (0) + mgh = E_{K,f} - (0)$$

Inserting known values, we can solve for the final kinetic energy:

$$\begin{aligned}(0.6\text{kg}) \times (10\text{N/kg}) \times (0.5\text{m}) &= E_{K,f} \\ 3\text{J} &= E_{K,f}\end{aligned}$$

The kinetic energy of the ball when it reaches the bottom of the incline is 3J.

Note: *Different parts of the ball are moving at different speeds, so we cannot determine the speed of the ball when it reaches the bottom.*